| Name ANSWERS | Class |
|---|---|

MATHS TEACHER HUB

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# Equation of a circle

## (9 – 1) Topic booklet

# Higher

**These questions have been collated from previous years GCSE Mathematics papers.**

**You must have:** Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

Total Marks

## Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out.**
- If the question is a **1H** question you are not allowed to use a calculator.
- If the question is a **2H** or a **3H** question, you may use a calculator to help you answer.

## Information

- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

**Answer ALL questions**
**Write your answers in the space provided.**
**You must write down all the stages in your working.**

**15** The equation of a circle is $x^2 + y^2 = 42.25$

Find the radius of the circle.

$r = \sqrt{42.25}$

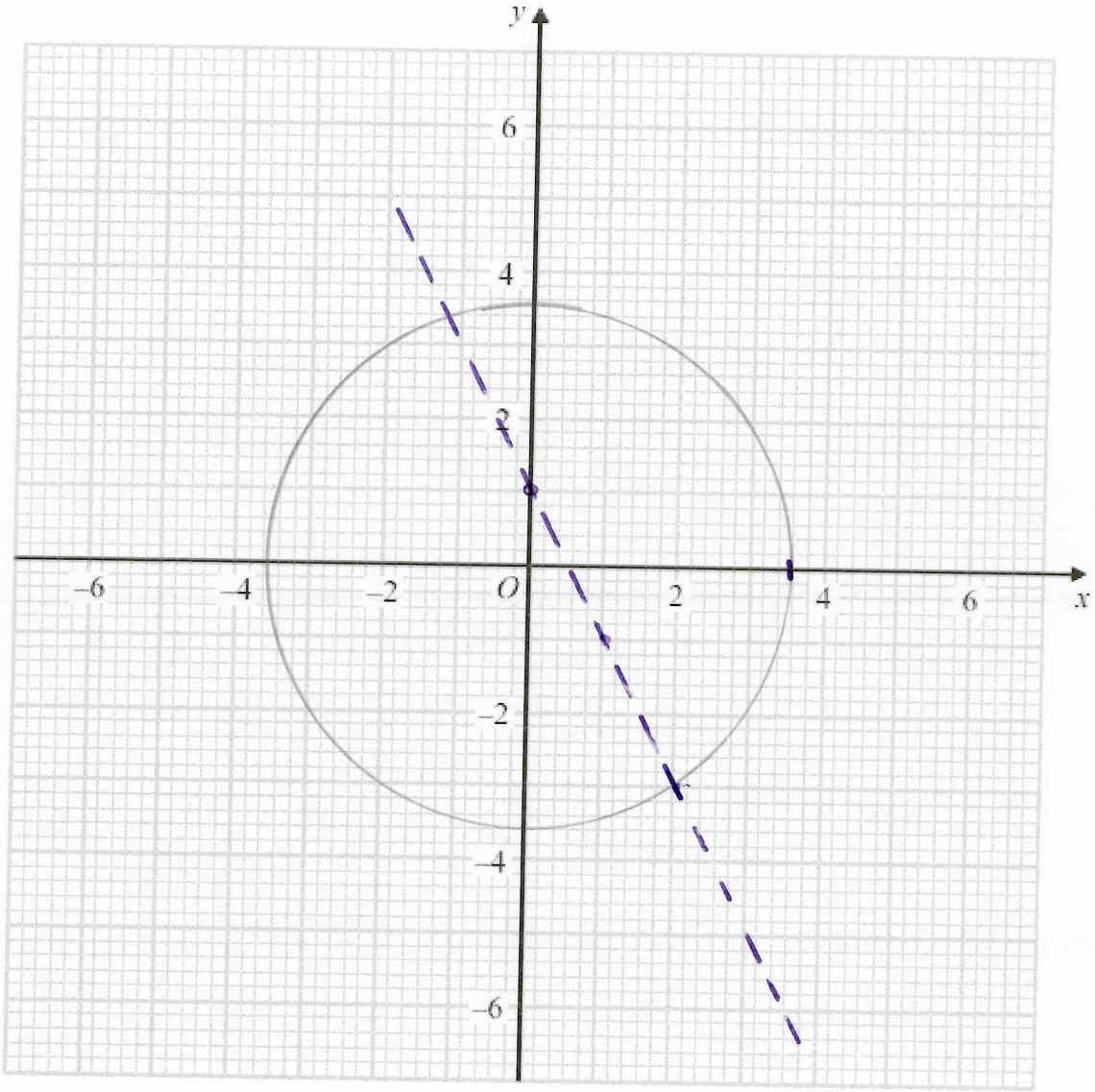
6.5



**(Total for Question 15 is 1 mark)**

**16** (a) On the grid, draw the graph of $x^2 + y^2 = 12.25$

$\sqrt{12.25} = 3.5$




(2)

(b) Hence find estimates for the solutions of the simultaneous equations

$$x^2 + y^2 = 12.25$$
$$2x + y = 1$$

$y = -2x + 1$

$x = 2 \quad y = -2.9$ , $x = -1.2 \quad y = 3.3$

(3)



**(Total for Question 16 is 5 marks)**

**19** Prove algebraically that the straight line with equation $x - 2y = 10$ is a tangent to the circle with equation $x^2 + y^2 = 20$

$x = 2y + 10$

$x^2 = (2y + 10)(2y + 10)$

$x^2 = 4y^2 + 40y + 100$

$4y^2 + 40y + 100 + y^2 = 20$

$5y^2 + 40y + 100 = 20$

$5y^2 + 40y + 80 = 0$

$\div 5$ $\quad y^2 + 8y + 16 = 0$

$(y + 4)(y + 4) = 0$

$\boxed{y = -4}$

$x - 2y = 10$

$x - 2(-4) = 10$

$x + 8 = 10$

$\boxed{x = 2}$

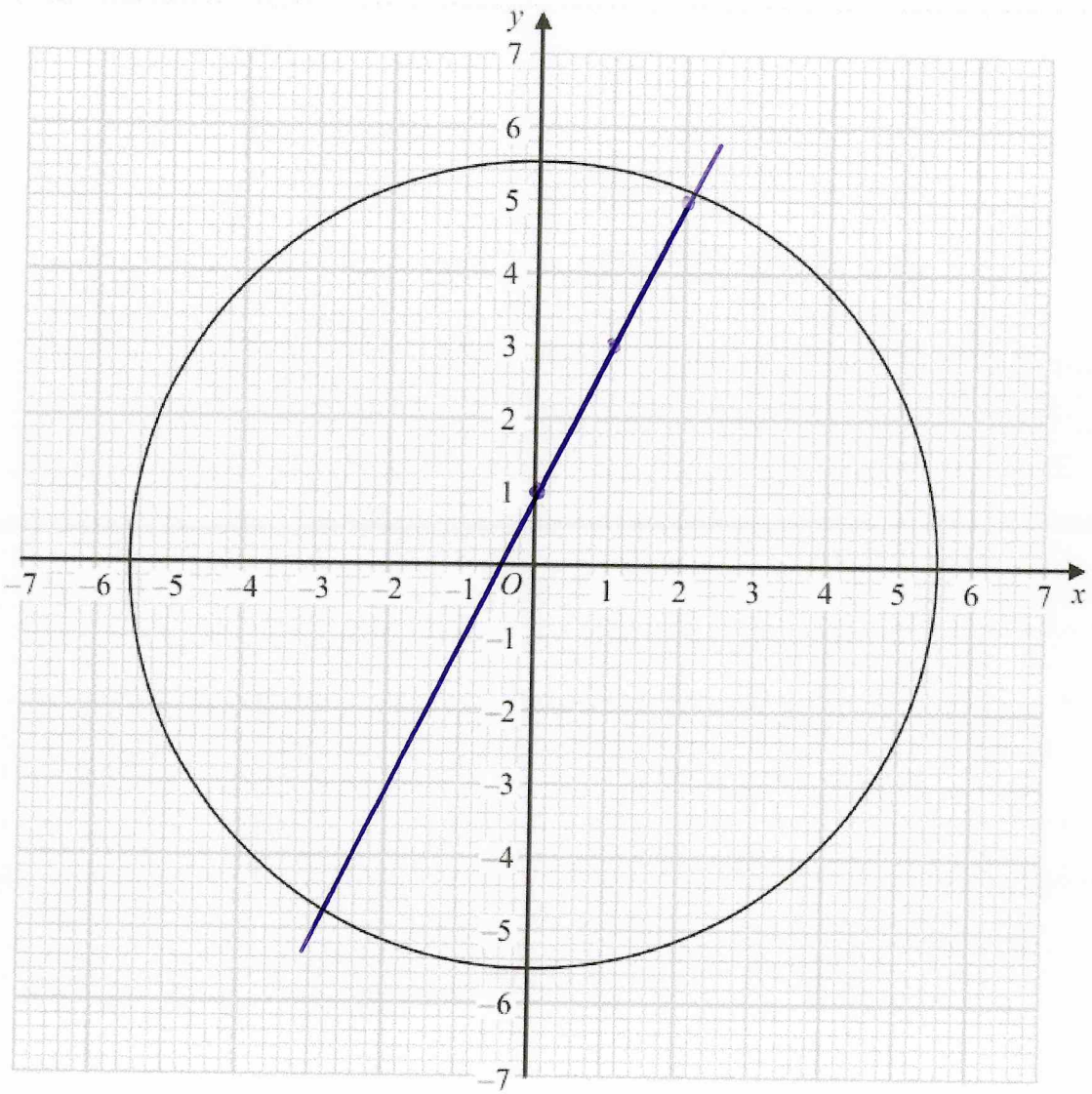
Point of intersection $= (2, -4)$

There is only 1 point of intersection, therefore the tangent and radius meet at one point only.



**(Total for Question 19 is 5 marks)**

**20** The diagram shows the graph of $x^2 + y^2 = 30.25$




Use the graph to find estimates for the solutions of the simultaneous equations

$x^2 + y^2 = 30.25$
$y - 2x = 1$

$y = 2x + 1$

$x = 2$ $y = 5$ , $x = -2.9$ $y = -4.7$

**(Total for Question 20 is 3 marks)**

**20** The equation of a curve is $y = a^x$
$A$ is the point where the curve intersects the $y$-axis.

(a) State the coordinates of $A$.

Crosses y when $x = 0$

$y = a^0$

$y = 1$

( 0 , 1 )

**(1)**

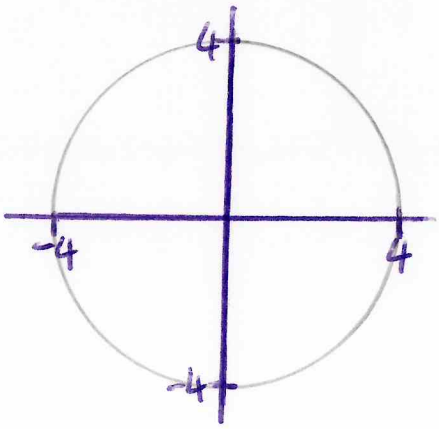
The equation of circle **C** is $x^2 + y^2 = 16$

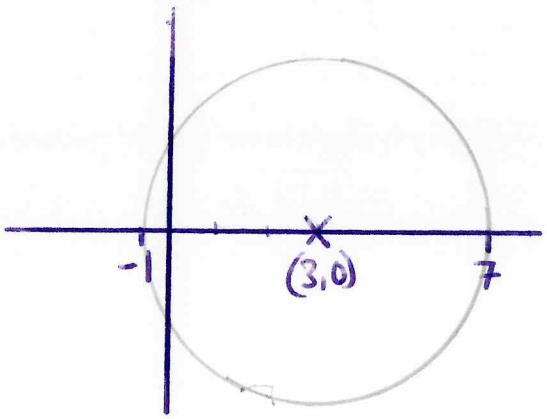
The circle **C** is translated by the vector $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ to give circle **B**.

(b) Draw a sketch of circle **B**.

$\sqrt{16} = 4$

Label with coordinates
the centre of circle **B**
and any points of intersection with the $x$-axis.



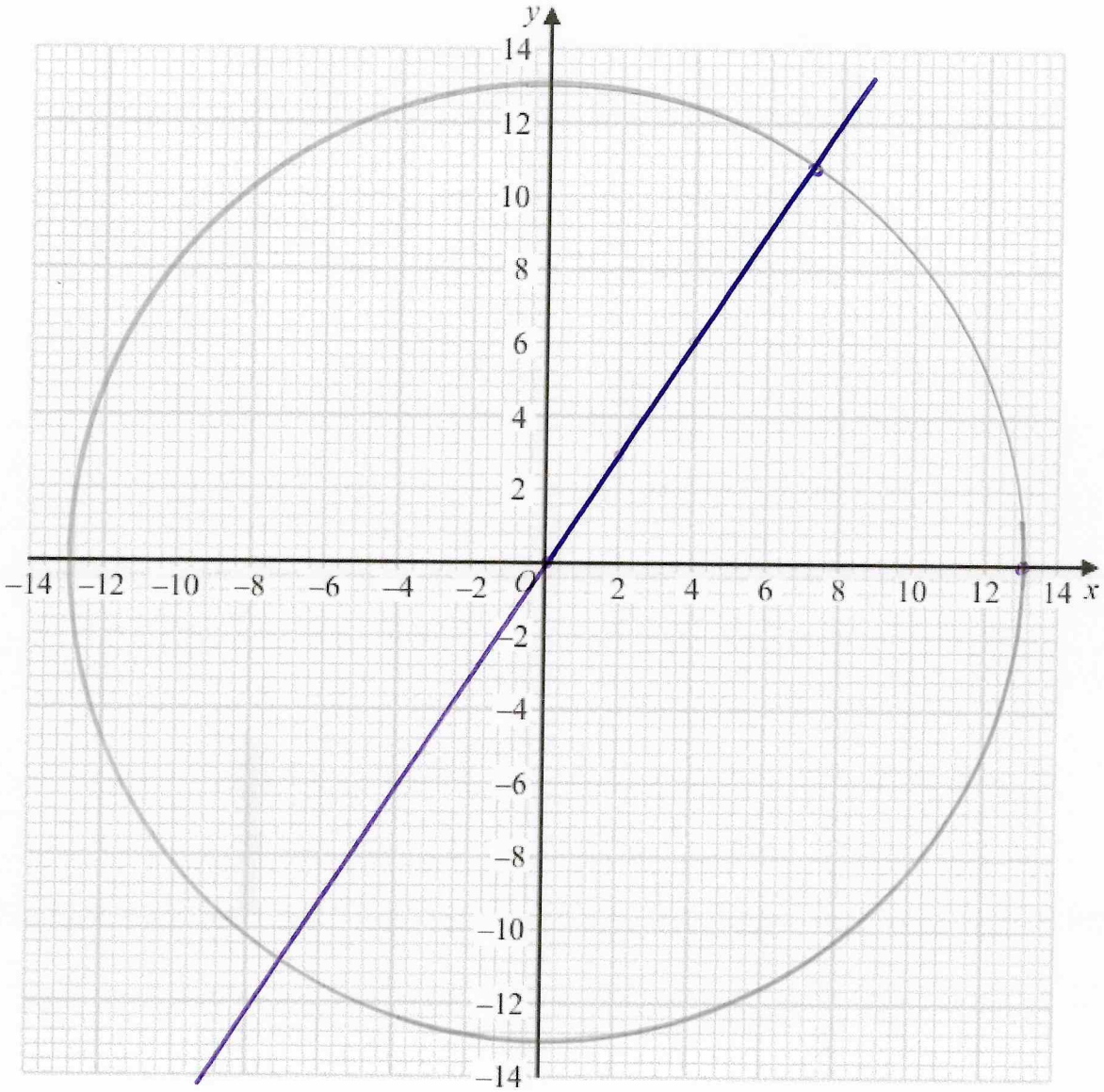

**(3)**



**(Total for Question 20 is 4 marks)**

**21** (a) On the grid, draw the graph of $x^2 + y^2 = 169$

$\sqrt{169} = 13$




(2)

(b) Use your graph to find estimates for the solutions of the simultaneous equations

$$x^2 + y^2 = 169$$
$$2y = 3x$$

$y = \frac{3}{2}x$

$x = 7.2$ $y = 10.8$, $x = -7.2$ $y = -10.8$

(3)

**(Total for Question 21 is 5 marks)**

**22** **C** is a circle with centre the origin.

A tangent to **C** passes through the points (–20, 0) and (0, 10)

Work out an equation of **C**.
You must show all your working.

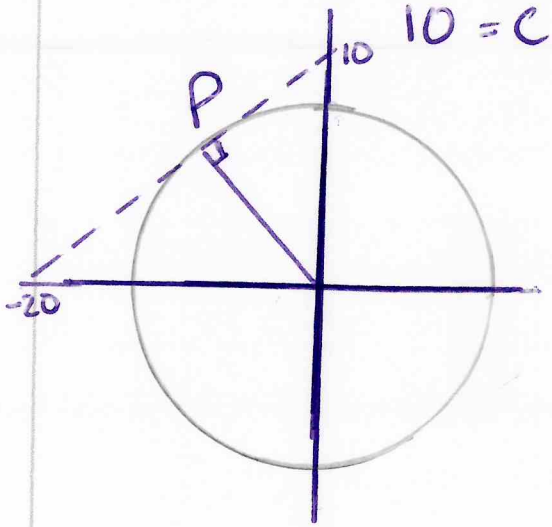
Tangent $m = \frac{\Delta y}{\Delta x} = \frac{10-0}{0--20} = \frac{10}{20} = \frac{1}{2}$

$y = \frac{1}{2}x + c$

$10 = \frac{1}{2}(0) + c$

$10 = c$

$y = \frac{1}{2}x + 10$




Radius $m = -2x$

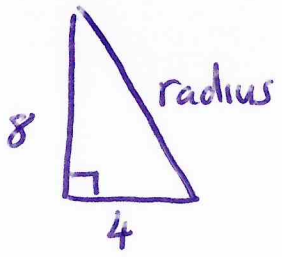
$y = -2x$

$-2x = \frac{1}{2}x + 10$

$-10 = 2\frac{1}{2}x$

$-4 = x$

$P = (-4, 8)$




$4^2 + 8^2 = r^2$

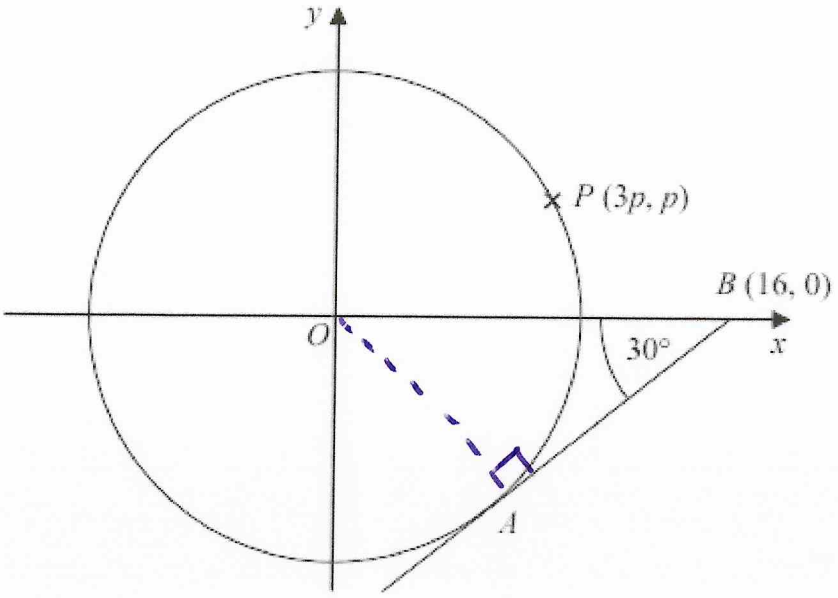
$16 + 64 = r^2$

$80 = r^2$

$x^2 + y^2 = 80$

**(Total for Question 22 is 5 marks)**

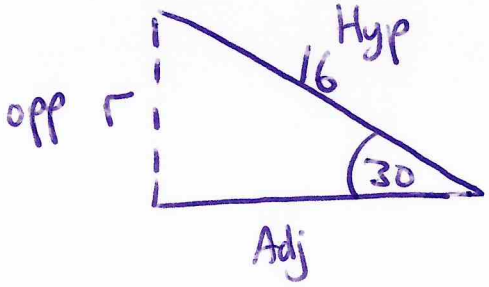
**22** The diagram shows a circle, centre $O$.




$AB$ is the tangent to the circle at the point $A$.
Angle $OBA = 30°$

Point $B$ has coordinates $(16, 0)$
Point $P$ has coordinates $(3p, p)$

Find the value of $p$.
Give your answer correct to 1 decimal place.
You must show all your working.




$\sin 30 = \frac{r}{16}$

$16 \times \sin 30 = \text{radius}$

$8 = r$

$x^2 + y^2 = 8^2$

$x^2 + y^2 = 64$

$(3p)^2 + (p)^2 = 64$

$9p^2 + p^2 = 64$

$10p^2 = 64$

$p = \sqrt{6.4}$

$p =$ 2.5

**(Total for Question 22 is 4 marks)**

**22** The line $l$ is a tangent to the circle $x^2 + y^2 = 40$ at the point $A$.
$A$ is the point (2, 6).

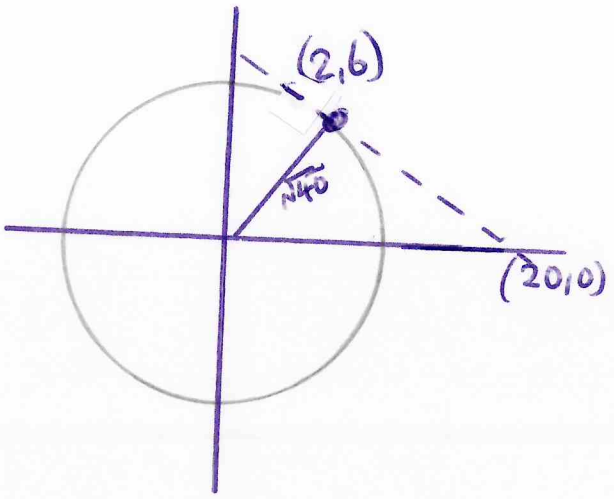
The line $l$ crosses the $x$-axis at the point $P$.

Work out the area of triangle $OAP$.

$x^2+y^2=r^2$

$40=r^2$

$\sqrt{40}=r$




m of radius $= \frac{\Delta y}{\Delta x} = \frac{6-0}{2-0} = 3$

m of tangent $= -\frac{1}{3}$

(2,6) $y = -\frac{1}{3}x + c$

$6 = -\frac{1}{3}(2) + c$

$6 = -\frac{2}{3} + c$

$6\frac{2}{3} = c$

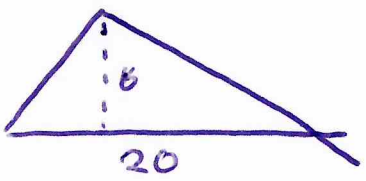
$\boxed{y = -\frac{1}{3}x + \frac{20}{3}}$

$y=0$

$0 = -\frac{1}{3}(x) + \frac{20}{3}$

$x = 20$

Area




$\frac{20\times6}{2} = \frac{120}{2} = 60$

60



**(Total for Question 22 is 5 marks)**

**23** **L** is the circle with equation $x^2 + y^2 = 4$

$P\left(\frac{3}{2}, \frac{\sqrt{7}}{2}\right)$ is a point on **L**.

Find an equation of the tangent to **L** at the point $P$.

m of radius $= \frac{\Delta y}{\Delta x} = \frac{\frac{\sqrt{7}}{2} - 0}{\frac{3}{\sqrt{2}} - 0} = \frac{\sqrt{7}}{3}$

m of tangent $= -\frac{3\sqrt{7}}{7}$

$y = -\frac{3\sqrt{7}}{7}(x) + c$

$y = -\frac{3\sqrt{7}}{7}\left(\frac{3}{2}\right) + c$

$\frac{\sqrt{7}}{2} = -\frac{9\sqrt{7}}{14} + c$

$\frac{\sqrt{7}}{2} + \frac{9\sqrt{7}}{14} = c$

$\frac{8\sqrt{7}}{7} = c$

$y = -\frac{3\sqrt{7}}{7}x + \frac{8\sqrt{7}}{7}$



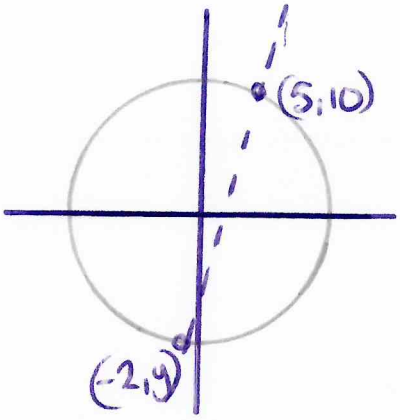
**(Total for Question 23 is 3 marks)**

**23** **C** is a circle with centre (0, 0)
**L** is a straight line.

The circle **C** and the line **L** intersect at the points $P$ and $Q$.
The coordinates of $P$ are (5, 10)
The $x$ coordinate of $Q$ is –2

**L** has a positive gradient and crosses the $y$-axis at the point $(0, k)$

Find the value of $k$.




$x^2+y^2=r^2$

$5^2+10^2=r^2$

$125=r^2$

$(-2)^2+y^2=125$

$4+y^2=125$

$y^2=121$

$y=\pm 11$

$y=-11$

$(5,10)\quad(-2,-11)$

$m=\frac{\Delta y}{\Delta x}=\frac{-11-10}{-2-5}=\frac{-21}{-7}$

$m=3$

$y=3x+c$

$10=3(5)+c$

$10=15+c$

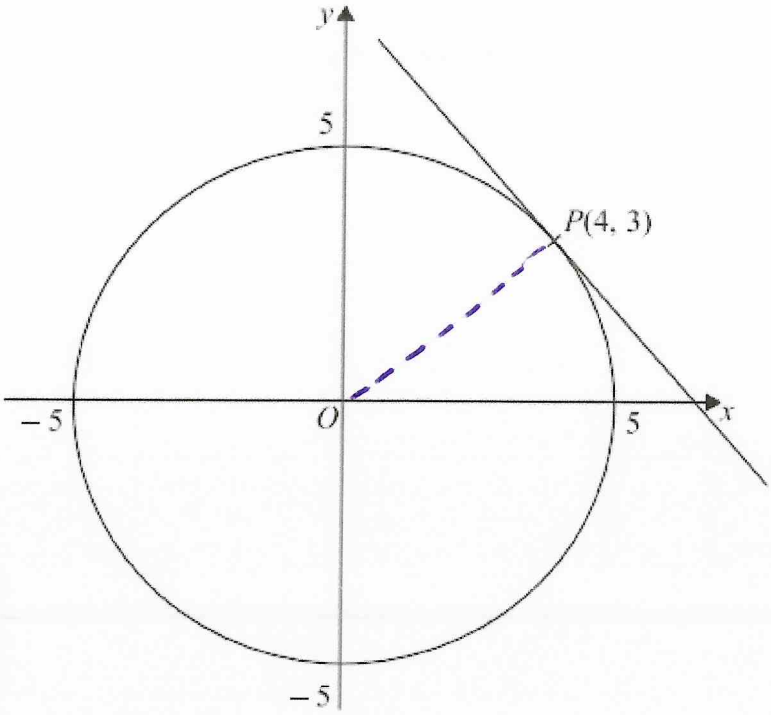
$-5=c$

$k=$ $-5$



**(Total for Question 23 is 5 marks)**

**23** Here is a circle, centre $O$, and the tangent to the circle at the point $P(4, 3)$ on the circle.




Find an equation of the tangent at the point $P$.

m of radius $= \frac{\Delta y}{\Delta x} = \frac{3-0}{4-0} = \frac{3}{4}$

m of tangent $= -\frac{4}{3}$

$y = -\frac{4}{3}x + c$

$3 = \frac{-4}{3}(4) + c$

$3 = \frac{-16}{3} + c$

$8\frac{1}{3} = c$

$y = \frac{-4}{3}x + 8\frac{1}{3}$

**(Total for Question 23 is 3 marks)**

23 A circle has equation $x^2 + y^2 = 25$

$\sqrt{25} = 5$

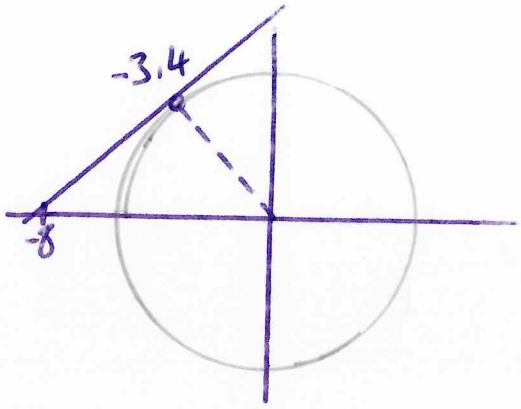
The point $P$ with coordinates $(-3, 4)$ lies on the circle.

Alex says that the tangent to the circle at $P$ crosses the $x$-axis at the point $(-8, 0)$

Is Alex correct?
You must show how you get your answer.

Radius

$$m = \frac{\Delta y}{\Delta x} = \frac{4-0}{-3-0} = \frac{4}{-3}$$




Tangent

$$m = \frac{3}{4}$$

$$y = \frac{3}{4}x + c$$

$(-3,4)$ $4 = \frac{3}{4}(-3) + c$

$4 = \frac{-9}{4} + c$

$c = 6\frac{1}{4}$

$$y = \frac{3}{4}x + 6\frac{1}{4}$$

Crosses $x$ when $y = 0$

$0 = \frac{3}{4}x + 6\frac{1}{4}$

$0 = 3x + 25$

$-25 = 3x$

$\frac{-25}{3} = x$

Alex is not correct, the tangent crosses at $-8\frac{1}{3}$ not $-8$.



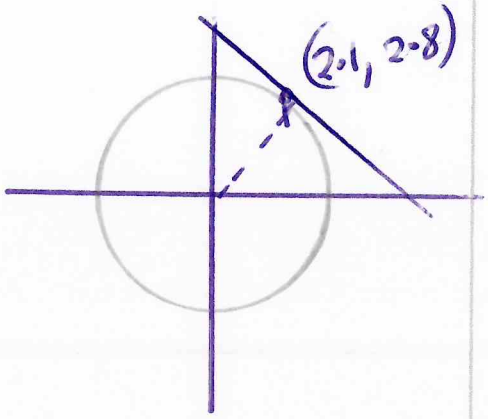
**(Total for Question 23 is 4 marks)**

**24** A circle has equation $x^2 + y^2 = 12.25$

The point $P$ lies on the circle.
The coordinates of $P$ are (2.1, 2.8)

The line **L** is the tangent to the circle at point $P$.

Find an equation of **L**.
Give your answer in the form $ax + by = c$, where $a$, $b$ and $c$ are integers.




Radius

$$m = \frac{\Delta y}{\Delta x} = \frac{2.8-0}{2.1-0} = \frac{4}{3}$$

Tangent

$$m = -\frac{3}{4}$$

$$y = -\frac{3}{4}x + c$$

$$2.8 = -\frac{3}{4}(2.1) + c$$

$$2.8 = -\frac{63}{40} + c$$

$$2.8 + \frac{63}{40} = c$$

$$4\frac{3}{8} = c$$

$$\boxed{y = -\frac{3}{4}x + 4\frac{3}{8}}$$

$$y = -\frac{3}{4}x + 4\frac{3}{8}$$

$$8y = -6x + 35$$

$$6x + 8y = 35$$

$6x + 8y = 35$



**(Total for Question 24 is 4 marks)**